Guitar Body Volume

(using tetrahedral approximation)

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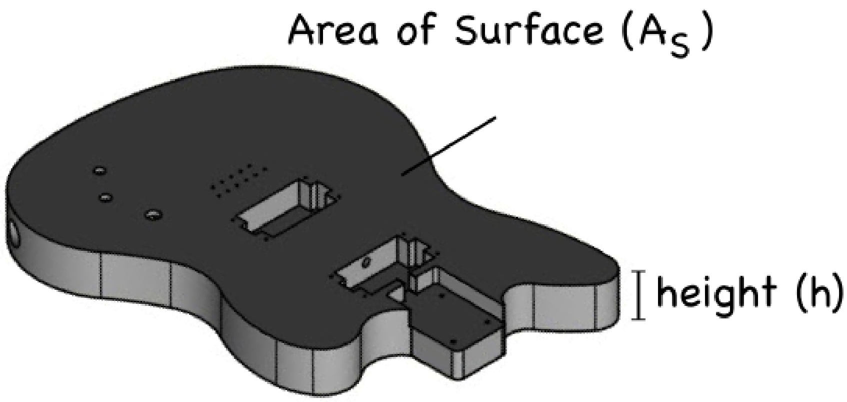
Abstract

Determining the surface area and volume of an electric guitar's body is the initial step in studying the effect of the size and shape of the body on the sound. We outline the guitar body's shape on a grid and plot points along the outline. By summing the areas of the small tetrahedra made on the grid we determine the total surface area. This project is designed to be a precursor to introducing the concept of limits in calculus while reviewing subscript and summation notation in equations.

1 Objective: Find the volume of a guitar body

The size, shape, mass and center of gravity of a electric guitar's body can potentially affect the sound because they affect the way the strings vibrate and the way that the body resonates with those vibrations. How exactly it affects the sound we're not sure about yet. We'll figure that out later when we attach vibration sensors to the body. However, as a first step, we'll work on the size and calculate the volume of the guitar.

Figure 1: *Calculating the volume of a guitar. (Image via guitarbuilding.org)*



The volume of any shape with parallel sides can be calculated by the surface area (A_S) times the height (h):

$$V = A_S\, h \tag{1}$$

where:

- V - volume [L^3]
- A_S - Area of Surface [L^2]
- h - height/thickness of guitar [L]

We'll assume that the guitar is of uniform thickness, even if you've already sculpted your guitar, so finding the height is easy, you can just measure it. So, the key challenge is to find the surface area.

Since the guitar's outline is not a common geometric shape, we'll have to approximate the surface area.

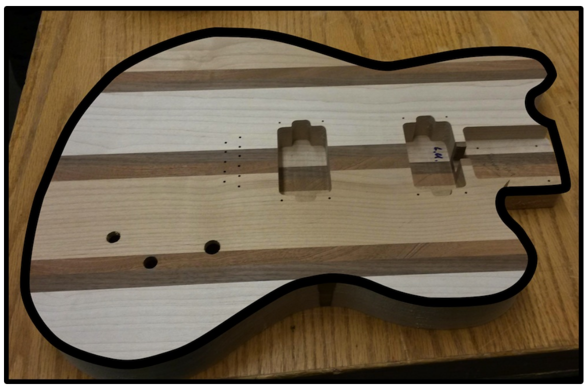
Detailed instructions follow, but if you're feeling ambitious skip to Figure ?? and use that method.

2 Finding the Surface Area

2.1 Tracing the Outline

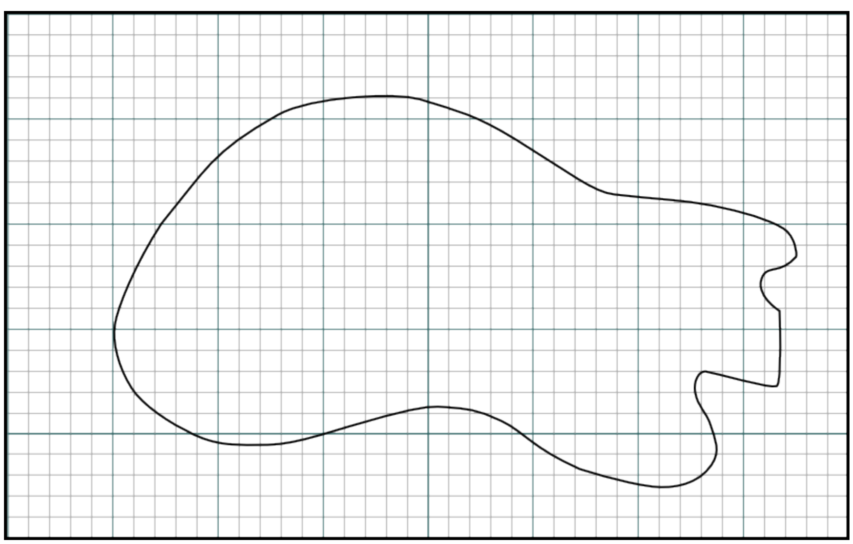
First we need to make an outline of the shape. You can lay the guitar out on a large piece of tracing paper and trace with a pencil, or take a picture (top down) and trace the edges on the computer.

Figure 2: *Tracing the outline. This picture was taken at an angle, not from straight up and down, so it's not a good example.*



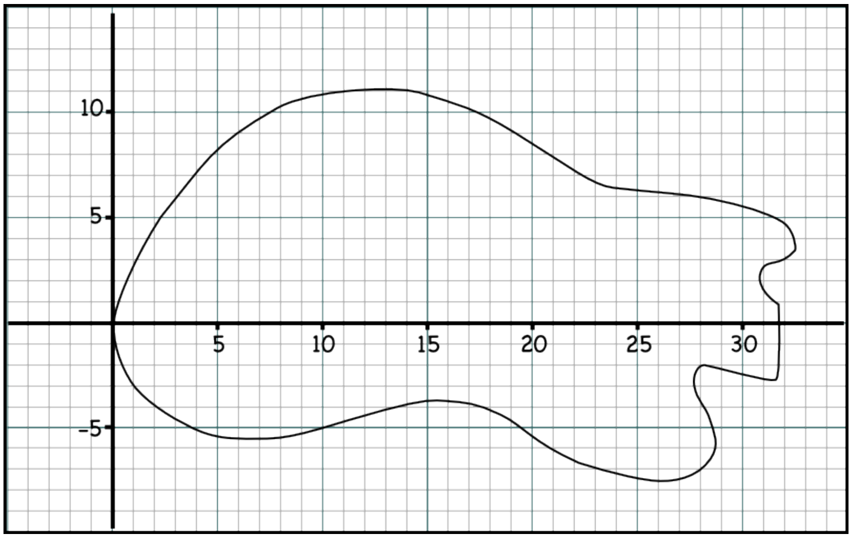
Now we lay the outline on grid paper. You can rotate the outline to align better with the grid.

Figure 3: *Placing on the grid. Notice that the outline here is slightly rotated.*



The next step is to define a set of axes. I've rotated the outline and set the zero point at the far edge.

Figure 4: *Choosing the axes.*

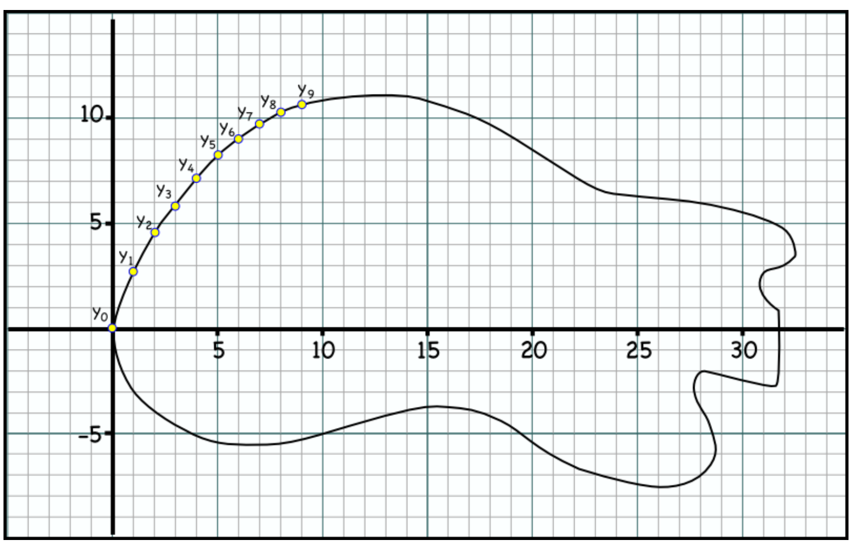


2.2 Area Using Tetrahedra

There are a number of different ways of finding the area of the outline, but we're going to make tetrahedral approximations using points where the outline intersects the grid.

First plot points.

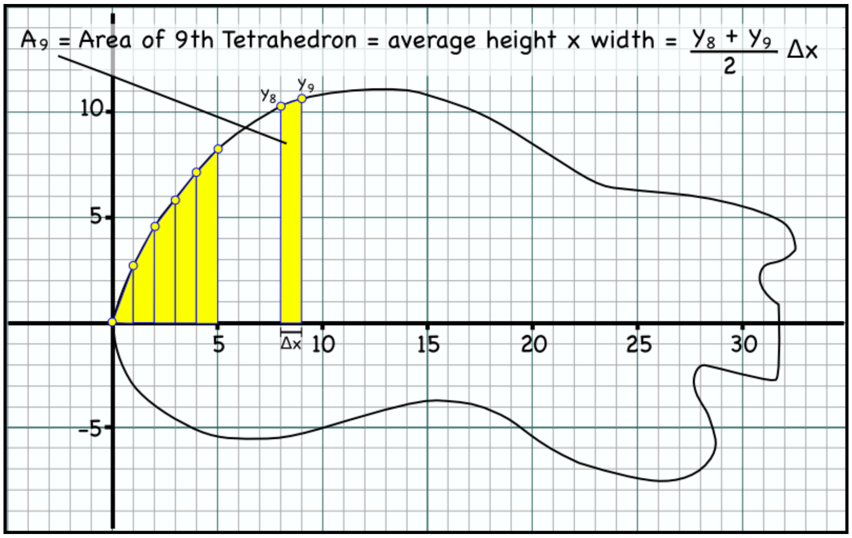
Figure 5: *Plotting points along the outline. Only the first nine points are plotted for demonstration.*



Now draw tetrahedra with sides parallel to the y-axis and perpendicular to the x-axis along each grid line (or choose your own spacing) and calculate the total area of the shape.

The area of each tetrahedron is the average height of the shape times the width. So, the area of

Figure 6: *Finding the area of the tetrahedra.*



the first tetrahedron is:

$$A_1 = \frac{y_0 + y_1}{2}\,\Delta x \tag{2}$$

And the area of the 9th tetrahedron (as shown in Figure **??**) is:

$$A_9 = \frac{y_8 + y_9}{2}\,\Delta x \tag{3}$$

In this example, we can read the values off the graph in Figure **??**,

- $y_8 = 10.2$
- $y_9 = 10.7$
- $\Delta x = 1.0$

So the area of the 9th tetrahedron can be calculated:

$$A_9 = \frac{y_8 + y_9}{2}\,\Delta x$$
$$A_9 = \frac{10.2 + 10.7}{2}\,1.0$$
$$A_9 = \frac{0.5}{2}$$
$$A_9 = 0.25$$

2.2.1 A General Equation for the Area

Looking at the two equations (Eqns. **??** & **??**), we should be able to make out the pattern in how specific points are related to specific areas. If you don't see the pattern immediately, writing out the equations for the first five tetrahedrons in a table should help. Based on the pattern we observe, we can write a general equation for the area using subscript notation:

$$A_i = \frac{y_{i-1} + y_i}{2}\,\Delta x \tag{4}$$

This is important so make absolutely sure you know how we came up with this equation.

As an aside, an alternative way of writing the equation, using function notation, is:

$$A(x) = \frac{f(x) + f(x+\Delta x)}{2}\,\Delta x \tag{5}$$

(However, as you'll realize that for this example at least, when you get to the right side of the shape, the outline does not pass the vertical line test, so it's not function. We'll see some elegant ways of dealing with functions later.)

2.2.2 Finding the Total Area

And finally, since we add all the areas together to get the total area, we can write an equation for the total area of the surface using sigma notation:

$$A_S = \sum_{i=1}^{n} A_i \tag{6}$$

which becomes (when we substitute in Equation **??**):

$$A_S = \sum_{i=1}^{n} \frac{y_{i-1} + y_i}{2}\,\Delta x \tag{7}$$

To clean up a bit, since we're using a constant value for the width (Δx) and the 2 on the denominator is also a constant, we can factor them out of the summation to get:

$$A_S = \frac{\Delta x}{2} \sum_{i=1}^{n} (y_{i-1} + y_i) \tag{8}$$

Now doing all this will take some time, but it would be a lot easier if you put the points you measure into a spreadsheet program (like Excel) and do the calculations there.

3 The Important Follow-up Questions

1) Your final answer is an approximation. What adjustments can you make to this method to get a more accurate result?

2) Making the width of your tetrahedra (Δx) smaller would have given less error. Would this change the general equation for tetrahedra (Eqn. **??**) or the equation for the total area (Eqn. **??**)?